THE NATIONAL TECHNICAL UNIVERSITY OF UKRAINE
"Igor Sikorsky Kyiv Polytechnic Institute"

Department of Mathematical Physics and Differential Equations

HIGHER MATHEMATICS PART 1.
Differential and Integral Calculus of a Function of One Variable.
SYLLABUS

Description of the Discipline	
Level of HE	*The first bachelor*
Branch of Knowledge	*13 Mechanical engineering*
Specialty	*131 "Applied Mechanics"*
Educational Program	*Manufacturing Engineering*
Status of the Discipline	*Compulsory.*
Form of Study	*Full-time*
Year and Semester	*First year, autumn semester*
Number of Credits	*4,5 credits ECTS (135 hours), including 18 hours of Lectures, 36 hours pf Practical Lessons, 81 hours of Self-study work*
Semester's Control	*Exam/ Midterm test, Individual tasks*
Schedule	*According to the schedule on the university website*
Language	*English*
Information About Course Teacher	*Ganna Zhuravska, Ph.D., Associate Professor* https://mph.kpi.ua/osobovij-sklad.html
Learning Resources	*Learning resources are determined by the course teacher and are given to students in on the first lesson. They consist of links to remote resources in Moodle, Google classroom, information resources in the university library, on the website of the department, etc.*

Course Description

1. Learning Objectives, Tasks and Outcomes of the Discipline

The discipline "Higher Mathematics Part 1. Differential and integral calculus of a function of one variable" is the first credit module belongs to the discipline "Higher Mathematics", which is included in the cycle of natural-scientific trainings for bachelors of the specialty 131 "Applied Mechanics".

The purpose of the discipline is studying the basic concepts and methods of the theory of limits, differential and integral calculus of the function of one variable; developing of the student's capability to use theory for solving typical problems on these topics; application of acquired knowledge, skills and abilities to solve applied problems of mathematics, mechanics, physics and in their daily practice; independent usage and studying of mathematical literature and other information sources.

Objective of the discipline is shaping the following knowledge and skills of students.

According to the matrix of correspondence of program competencies

- *general competencies:*

GC1. Ability to abstract thinking, analysis and synthesis;

- *professional competencies:*

PC1. Ability to analyze materials, structures and processes based on laws, theories and methods of mathematics, natural sciences and applied mechanics.

Students must demonstrate the following learning outcomes.

According to the matrix of correspondence of program learning outcomes in the educational program

- LO1) Ability to choose and apply convenient mathematical methods for solving problems of applied mechanics.
- **Knowledge:** basic definitions of the functions of one variable (domain, range, basic characteristics of functions, basic elementary functions and their graphs); the principles of differential calculus of functions of one variable (limit of a numerical sequence, limit of a function, concept of infinitesimals, concept of continuous function, the points of discontinuity of function, the tangent and the normal line to the curve, the derivative and the differential of a function, asymptotes, applications of derivatives: monotonicity, extrema and concavity, L'Hospital's rule); the bases of the integral calculus of functions of one and many variables (indefinite integral, properties of an indefinite integral, a table of integrals, methods of integration).
- **Skills:** to know of graphs of basic elementary functions, to perform transformations of graph, to find the roots of polynomials, factoring polynomials, to perform operations on complex numbers; to find the limits of sequences and functions, to compare infinitesimals, to investigate the function for continuity, to classify points of discontinuity and asymptotes of the function, to find derivatives and differentials of functions of one variable, to apply the derivative and differential calculus, to make the investigation of functions, to use of L'Hospital's rule; to find indefinite integrals by basic methods of integration.
- **Experience:** thinking logically and flexibly; independently using and studying literature on mathematics; generalizing, setting the goal and choosing ways to solve it; using methods of mathematics in engineering calculations.

2. Prerequisites and Postrequisites of the Discipline (place in the structural and logical scheme of education according to the relevant educational program)

Interdisciplinary connections:

The discipline "Higher Mathematics Part 1. Differential and integral calculus of a function of one variable" is closely related to the disciplines "Higher Mathematics Part 2. Differential and integral calculus of many variables. Differential Equations" and "Higher Mathematics Part 3. Series. Theory of the function of a complex variable", which are studied in the following semesters. This discipline provides other disciplines of the natural sciences

cycle such as Physics, Engineering Graphics, Computer Science, Electrical Engineering and Electronics; must precede and provide the following disciplines in the program of professional and practical training of a specialist: Engineering Mechanics, *Strength of Materials,* Technology of Structural Materials, Theory of Mechanisms and Machines, Theoretical Mechanics, Mathematical Modeling and other disciplines (according to the structural and logical scheme of the educational program).

3. Content of the Discipline

Part 1. Theory of Limits.

Topic 1.1. The Concept of Sets. Binary Operations with Sets. The Set of Real Numbers and Their Properties. The Absolute Value of a Real Number.

Topic 1.2. The Concept of Function. Representing of Functions. Properties of Functions. The Inverse Function. Composite Function. Basic Elementary Functions and Their Graphs.

Topic 1.3. Numerical Sequence. Basic Concepts. The Limit of a Numerical Sequence. Number **e**. Natural Logarithms.

Topic 1.4. The Limit of a Function. One-sided Limits. Infinitely Large Functions. Infinitesimals and Their Properties.

Topic 1.5. Basic Theorems on Limits. The First Important Limit. The Second Important Limit.

Topic 1.6. Comparing of Infinitesimals. Equivalent Infinitesimals and Their Applications.

Topic 1.7. Continuity of Functions. Classification the Points of Discontinuity of a Function. Basic Theorems on Continuous Functions.

Topic 1.8. Certain Properties of Continuous Functions.

Part 2. Differential Calculus of a Function of One Variable.

Topic 2.1. Definition of Derivative. Mechanical, Physical and Geometric Meanings of the Derivative. The Equations of Tangent and Normal Lines. Differentiability of Functions. Basic Rules of Differentiation: Derivative of a Sum, a Product, and a Quotient. The Derivative of a Composite Function (Chain Rule). The Derivative of an Inverse Function.

Topic 2.2. Derivatives of Basic Elementary Functions: Power Function, Exponential Function, Logarithmic Function, Trigonometric Functions, Inverse Trigonometric Functions, Hyperbolic Functions. The Table of Derivatives.

Topic 2.3. The Derivative of a Function Represented Parametrically. The Derivative of an Implicit Function. The Logarithmic Differentiation.

Topic 2.4. Derivatives of Higher Orders. Mechanical Significance of the Second Derivative. Derivatives of Higher Orders of Implicit Functions and of Functions Represented Parametrically.

Topic 2.5. The Differential and Its Geometric Significance. Differentials of Higher Orders. Applications of Differentials.

Topic 2.6. Basic Theorems of the Differential calculus. A Theorem on the Roots of a Derivative. A Theorem on Finite Increments. A Theorem on the Ratio of the Increments of Two Functions. The L'Hospital's Rule.

Topic 2.7. Taylor's and Maclaurin's Formulas. Maclaurin's formulas for Basic Elementary Functions.

Topic 2.8. The Monotonicity of a Function. Local Extrema of a Function. The Largest and Smallest Values of a Function.

Topic 2.9. Concavity of a Curve. Points of Inflection. Asymptotes. The General Plan for Investigating Functions and Constructing Graphs.

Part 3. Integral Calculus of Functions of one variable.

Topic 3.1. The Concept of an Antiderivative. The Indefinite Integral and its properties. Table of Integrals.

Topic 3.2. Basic methods of Integration: Integration by Substitution. Integration by Parts.

4. Informational Support

Readings

1. H. Jerome Keisler Elementary Calculus: an Infinitesimal Approach/ H. Jerome Keisler – On-line Edition. 2000 https://www.math.wisc.edu/~keisler/calc.html
2. Higher Mathematics. Differential Calculus of a Function of One Variable. Elements of Theory [Electronic resource] / Igor Sikorsky Kyiv Polytechnic Institute ; compiler Ganna V. Zhuravska. – Electronic text data (1 file: 1,86 Mb). – Kyiv : Igor Sikorsky Kyiv Polytechnic Institute, 2019. – 81 p. https://ela.kpi.ua/handle/123456789/27234
3. Higher Mathematics. Integral Calculus of a Function of One Variable. Elements of Theory [Electronic resource] / Igor Sikorsky Kyiv Polytechnic Institute ; comp. G. V. Zhuravska. – Electronic text data (1 file: 1,31 Mb). – Kyiv : Igor Sikorsky Kyiv Polytechnic Institute, 2019. – 68 p. https://ela.kpi.ua/handle/123456789/27854
4. Swokowski, Earl William Calculus: 5th ed / Swokowski, Earl William - Published 1991 by Pws-Kent Publishing Company – ISBN 0-534-92492-1 - 1053 p.

Educational Content

5. Learning Activities and Teaching Methods

Basic teaching methods:

1. Providing during lectures theoretical material and examples of solving the main practical problems.

2. Solving examples and tasks during practical trainings through active and collective learning, part-search and research methods;

3. Self-studying of students, such as homework to each topic of the course and individual tasks.

4. As an addition to traditional training activities there are proposed such activities as internet mailing, distance and online learning, online testing etc.

5. Assessment is carried out with the help of various control measures: midterm tests, mathematical dictation, classroom activities, homework, individual tasks and final exam.

Lectures

№	The title of the lecture and a list of the main subtopics and task for the self-study
1	Numerical Sequence. Basic Concepts. The Limit of a Numerical Sequence. An Infinitely Large Variable. The Limit of a Function. One-sided Limits. Infinitely Large Functions. Infinitesimals and Their Properties. Task for the Self-study: Topic 1.1 The Concept of Sets. Binary Operations with Sets. The Set of Real Numbers and Their Properties. The Absolute Value of a Real Number. Topic 1.2 The Concept of Function. Ways of Representing Functions. Properties of Functions. The Inverse Function. Composite Function. Basic Elementary Functions and Their Graphs.
2	Basic Theorems on Limits. The First Important Limit. The Second Important Limit.
3	Comparing of Infinitesimals. Equivalent Infinitesimals and Their Applications. Continuity of Functions. Classification the Points of Discontinuity of a Function. Basic Theorems on Continuous Functions. Task for the Self-study: Topic 1.8. Certain Properties of Continuous Functions.
4	Definition of Derivative. Differentiability of Functions. Basic Rules of Differentiation: Derivative of a Sum, a Product, and a Quotient. The Derivative of a Composite Function (Chain Rule). The Derivative of an Inverse Function. The Derivative of a Function Represented Parametrically. The Derivative of an Implicit Function. The Logarithmic Differentiation. Task for the Self-study: Topic 2.1. Mechanical, Physical and Geometric Meanings of the Derivative. The Equations of a Tangent and a Normal Lines. Topic 2.2. Derivatives of Basic Elementary Functions: Power Function, Exponential Function, Logarithmic Function, Trigonometric Functions, Inverse Trigonometric Functions, Hyperbolic Functions. The Table of Derivatives.

5	Derivatives of Higher Orders. Mechanical Significance of the Second Derivative. Derivatives of Higher Orders of Implicit Functions and of Functions Represented Parametrically. The Differential and Its Geometric Significance. Differentials of Higher Orders.
6	Basic Theorems of the Differential calculus. A Theorem on the Roots of a Derivative. A Theorem on Finite Increments. A Theorem on the Ratio of the Increments of Two Functions. The L'Hospital's Rule. The Monotonicity of a Function. Local Extrema of a Function. The Largest and Smallest Values of a Function. Task for the Self-study: Topic 2.7. Taylor's and Maclaurin's Formulas. Maclaurin's formulas for Basic Elementary Functions.
7	Concavity of a Curve. Points of Inflection. Asymptotes. The General Plan for Investigating Functions and Constructing Graphs.
8	The Concept of an Antiderivative. The Indefinite Integral and its properties. Table of Integrals.
9	Basic methods of Integration: Integration by Substitution, Integration by Parts. Examples.

Practical Trainings

№	The title of the practical training and a list of the main subtopics
1	The Concept of Function. Ways of Representing Functions. Properties of Functions. The Inverse Function. Composite Function. Basic Elementary Functions and Their Graphs.
2	Numerical Sequence. Basic Concepts. The Limit of a Numerical Sequence. An Infinitely Large Variable.
3	The Limit of a Function. Infinitely Large Functions. Infinitesimals and Their Properties.
4	The First Important Limit.
5	The Second Important Limit.
6	Comparing of Infinitesimals. Equivalent Infinitesimals and Their Applications.
7	Continuity of Functions. Classification the Points of Discontinuity of a Function.
8	Midterm Test 1.
9	Derivative. The Table of Derivatives. Basic Rules of Differentiation: Derivative of a Sum, a Product, and a Quotient. Chain Rule.
10	The Derivative of a Function Represented Parametrically. The Derivative of an Implicit Function. The Logarithmic Differentiation.
11	Derivatives of Higher Orders. Derivatives of Higher Orders of Implicit Functions and of Functions Represented Parametrically.

12	Differentials. The Equations of a Tangent and a Normal Lines.
13	Basic Theorems of the Differential calculus. The L'Hospital's Rule. Taylor's and Maclaurin's Formulas.
14	The Monotonicity and Local Extrema of a Function. The Largest and Smallest Values of a Function. Concavity of a Curve. Points of Inflection. Asymptotes. The General Plan for Investigating Functions and Constructing Graphs.
15	Midterm Test 2.
16	The Concept of an Antiderivative. The Indefinite Integral and its properties. Table of Integrals.
17	Basic methods of Integration: Integration by Substitution, Integration by Parts.
18	Midterm Test 3.

6. Self-studying of Students

There are 81 hours of self-study work of students. They include: 35 hours for learning lectures and doing homework, 6 hours for preparing to midterm tests, 10 hours for doing individual tasks and 30 hours for preparing for final exam.

Self-study work also carries out studying some topics of the discipline.

Self-study Work

№	The title of topic for the self-study	Hours
Part 1. Theory of Limits.		
1	Topic 1.1 The Concept of Sets. Binary Operations with Sets. The Set of Real Numbers and Their Properties. The Absolute Value of a Real Number.	2
2	Topic 1.2 The Concept of Function. Ways of Representing Functions. Properties of Functions. The Inverse Function. Composite Function. Basic Elementary Functions and Their Graphs.	3
3	Topic 1.8. Certain Properties of Continuous Functions.	2
Part 2. Differential Calculus of a Function of One Variable.		
4	Topic 2.1. Mechanical, Physical and Geometric Meanings of the Derivative. The Equations of a Tangent and a Normal Lines.	1
5	Topic 2.2. Derivatives of Basic Elementary Functions: Power Function, Exponential Function, Logarithmic Function, Trigonometric Functions, Inverse Trigonometric Functions, Hyperbolic Functions. The Table of Derivatives.	3
6	Topic 2.7. Taylor's and Maclaurin's Formulas. Maclaurin's formulas for Basic Elementary Functions.	2

Policies and Assessments

7. Policies of the Discipline

Class-meetings are held in auditoriums according to the schedule. In the case of online classes that should be provided by the relevant order of the university, lectures and practical trainings are engaged in online video communications (Zoom, Google-Meet, Skype).

Requirements and the system of assessment are announced to students in the first lesson.

Attendance

Students are expected to attend each class meeting, since during these lessons they study theoretical material and develop the skills needed to complete homeworks, individual tasks and middle tests. There are no penalty points for absence from lectures and practical classes.

Absence During Control

Failure to submit the scheduled time the individual tasks and systematic failure to do homeworks without a valid reason is punishable by penalty points, according to the Rating System. Absence from the middle test, if the reason is not documented, is not give you a chance to do it in another time.

Academic Integrity Policy

The policy and principles of academic integrity are defined in Section 3 of the Code of Honor of the National Technical University of Ukraine "Igor Sikorsky Kyiv Polytechnic Institute".

Read more: https://kpi.ua/code.

Norms of Ethical Behavior

Norms of ethical behavior of students and teachers are defined in Section 2 of the Code of Honor of the National Technical University of Ukraine "Igor Sikorsky Kyiv Polytechnic Institute".

Read more: https://kpi.ua/code.

8. Assessment of Learning Outcomes and Rating System

Execution ongoing control and assessment of student's knowledge are made due to classroom activities, middle tests and individual tasks, the purpose of which is to identify the level of assimilation of relevant topics.

The final test is an exam in the end of each semester. The 100-points rating system and the university scale are used to evaluate the results of the training.

The Rating System

1. The student rating of the credit module is calculated from 100 points, 60 of them are the starting scores. Starting scores (during the semester) consists of the points that the student receives for:

1) classroom activities (18 lessons);
2) homework;
3) three middle tests;
4) individual tasks.

2. Criteria For Scoring Points

2.1. Classroom Activities:

– student has a thorough knowledge and understanding of the content and a high level of competence in the processes and skills - 1 point;

– student has achieved a basic level of competence in the processes and skills - 0.5 points;

– student has no elementary knowledge and understanding of the content - 0 points.

The maximum of points for classroom activities during the semester is 14.

2.2. Homework

It includes completing tasks at home for each topic covered. Correctly completed and timely homework is assessed at 0.2 points.

The maximum of points for all homework is 0.2 ×15=3 points.

2.3. Midterm Test

There are three Midterm tests

- complete solutions (no less than 90% of the required information) – 9-10 points;
- solutions with minor inaccuracies (no less than 75% of the required information) -7-8 points;
- mistakes in an solutions (no less than 60% of the required information) - 4-6 points;
- no solutions (less than 60% of the required information) - 0 points.

Midterm test №1	Max points
Topic: “Theory of Limits” (2 hours)	10
Midterm test №2	
Topic: “Differential Calculus of a Function of One Variable” (2 hours)	10
Midterm test №3	
Topic: “Integral Calculus of a Function of One Variable” (2 hours)	10

Use of a cell phone or any other communication device is not permitted during midterm tests. Violation of this policy will result in a score of 0 (zero) in the midterm tests.

Make-ups of the midterm test is allowed only in the case of a valid reason.

In the case of online learning, the midterm test is written by students in practical classes using Zoom or Skype platforms (or another, depending on the agreement with the teacher).

Students are sent the card for the test, and they, in two academic hours, must send photocopies of written solution via messenger (Telegram, What's Up etc.) or by e-mail. If the student's work is not sent in time, it is considered that the student was absent from the test, the work is not checked, and student receives 0 points.

2.4. Individual Tasks - 17 points.

- complete solutions – 15-17 points;
- solutions with minor inaccuracies - 12-14 points;
- mistakes in an solutions - 10-11 points;
- no solutions - 0 points.

Each week of delay of the submission of the individual tasks to examination are provided with penalty (-1) points.

The student must submit the completed individual tasks no later than the last lesson of the semester. In case of violation of this deadline, the student is considered not admitted to the exam of the main session. Student could be allowed to pass exam should submit the individual tasks before the additional session.

In the case of online learning, the individual tasks is checked by sending photocopies of the written work to the teacher's e-mail (or other platform, depending on the agreement with the teacher (Telegram, What's Up etc.)).

3. The requirement to the first midterm assessment (attestation) is to receive at least 11 points and to submit the corresponding part of individual tasks. The requirement to the second midterm assessment (attestation) is to receive at least 22 points and to submit the corresponding part of individual tasks.

4. The allowance requirement to passing the exam is the individual tasks, that must be credited, and no less than 36 points of starting score.

5. At the exam each student have two theoretical questions (8 points each) and three practical ones (8 points each). Total exam score is 40 points.

The criterion for evaluating theoretical questions:

- full answer (not less than 90% of the required information) - 8 points.
- sufficiently complete answer (not less than 75% of the required information, or minor inaccuracies) - 6-7 points.
- incomplete answer (not less than 60% of the required information and some mistakes) - 4-5 points.
- wrong answer or less than 60% of the required information - 0 points.

Criteria for evaluating practical tasks:

- complete solution of the task - 8 points.
- solution of the task with minor inaccuracies - 6-7 points.
- the task is solved with some mistakes - 4-5 points.
- the task is not solved - 0 points.

If the exam score is less than 24 points, then the student gets an "unsatisfactory" rating.

Use of books, a cell phone or any other communication device is not permitted during exams. Violation of this policy will result in a score of 0 (zero) in the exam.

6. The sum of the starting score and the exam score (personal score) is transferred into the grade of credit module according to the table:

PERSONAL SCORE	**GRADE**
95-100	Excellent
85-94	Very Good
75-84	Good
65-74	Satisfactory
60-64	Sufficiently
Personal score<60 or the exam score is less than 24 points	Unsatisfactory
Personal score < 36 or the individual tasks is not credited	Not allowed to pass exam

In the case of online learning, according to the teacher's decision, it is possible to set the exam grade by recalculating the starting points on a 100-point scale:

$$R = 60 + \frac{40(R_I - R_D)}{R_C - R_D},$$

where R_I - the amount of points earned by the student during the semester (starting score).

If the student does not agree with that grade, he takes the exam in video mode according to the schedule of the examination session.

Syllabus of the Discipline:

Compiled:

Inna Kopas, Ph.D., Associate Professor

Tamara Karpalyuk, Ph.D., Associate Professor

Tetiana Samoylenko, Ph.D., Assistant

Translated:

Ganna Zhuravska, Ph.D., Associate Professor

Approved:

Department of Mathematical Physics and Differential Equations (Protocol № 1 dated July 22, 2023)

Agreed:

Methodical Commission of Institute of Mechanical Engineering (Protocol №9 dated July 30, 2023)